**Homework 09: Theory of Angular Momentum**

**PHYS550 – Quantum Mechanics I**

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***Additional Texts Referenced: Introduction to Quantum Mechanics, Griffiths and Schroeter***

**Problem 3.1**

*Use the specific form of Sx given by (3.25) to evaluate (3.23) and show that Sx rotates as expected through an angle φ about the z-axis.*

First, taking 3.25 and turning it into matrix notation, we get:



3.23 is:



Which is the expected value for Jx. Let us replace Jx with Sx and work from there. As in the example for Jx, we use 2.168 to expand 3.23.



Note that J=L+S. So our Jz=Lz+Sz. So our commutator equals:



L and S are compatible observables, so that commutator goes to zero, leaving only the spin commutator. (This could be proven by splitting the J commutator into its L and S components and showing that the [L,S] parts have to go to zero to get the known answers.) Anyway, we may replace all the J operators with just their S components, giving us:



Note that this is now exactly the same as 3.24 just with S instead of J. Also note that the commutation relation for S, J, and L are all the same, following the pattern outlined for J in 3.20. We now get:



Which shows that, yes, it spins exactly as Jx did, which is to be expected.

**Problem 3.3**

*Find, by explicit construction using Pauli matrices, the eigenvalues for the Hamiltonian*

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*For a spin ½ particle in the presence of a magnetic field* ***B****=*

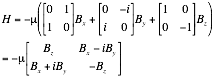
The Pauli matrices can be used to construct **S** like so:



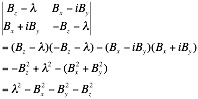
So if we perform the dot product we get:



Which would make the Hamiltonian:



This is the Hamiltonian. We note that it is Hermitian, which is a good sign. Now we do the general “find the eigenvalues” problem.



So naturally we have two eigenvalues:



Which is to say “the magnitude of the magnetic field, positive or negative.” (later multiplied by µ). As our matrix is 2D we should only have two eigenvalues, so this is all we need.

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Which can be thought of “we’re checking the spin based on the magnitude of the magnetic field.”